the srijan school

ASSIGNMENT
MATHEMATICS
CLASS XII

CHAPTER: DIFFERENTIAL EQUATIONS

Q1. Write order and degree of differential equation:

(i) $1 - \left(\frac{dy}{dx}\right)^2 = (y'')^{1/3}$

(ii) $y = px + \sqrt{a^2p^2 + b^2}\,,\ where\ p = \frac{dy}{dx}$.

Q.2. Form the differential equation of the following family of curves:

a. $x^2 + y^2 = 2ax$
b. $c(y + c)^2 = x^3$
c. **All lines , which are at a unit distance from the origin.**
d. **Family of circles passing through the fixed points (a, 0) and (- a , 0).**
e. **Family of circles in the second quadrant which touches the co-ordinate axis.**
f. **A spherical rain drop evaporates at a rate proportional to its surface area. Form a differential equation involving the rate of change of radius of the rain drop.**

Q. 3 Solve:

(i) $(cos\ x)\ \frac{dy}{dx} + cos\ 2x = cos\ 3x$ **(ii)** $\frac{dy}{dx} = 2xy = y$

(iii) $\frac{dy}{dx} = e^{x+y} + x^2e^y$ **(iv)** $\frac{dy}{dx} + sin(x + y) = sin(x - y)$

Q. 4 If $\frac{dy}{dx} + \frac{1+y^2}{1+x^2} = 0$ **,** $show\ that\ x + y = A\ (1 - xy)$**.**

Q. 5. Find the solution of the differential equation

$y - x\ \frac{dy}{dx} = a\left(y^2 + x^2\frac{dy}{dx}\right)$ **when x = a, y= a.**

Q. 6. Solve (i) $(x + y)^2\ \frac{dy}{dx} = a^2$ **(ii)** $\frac{dy}{dx} = sin\ (x + y) + cos\ (x + y) - 1$

Q. 7 Solve(i) $\left(x - \sqrt{xy}\ \right)dy = y\,dx$ **(ii)** $(x^2 + 3xy + y^2)dx - x^2\,dy = 0$.

(iii) $x\ \frac{dy}{dx} = y\ (log\ y - log\ x + 1)$ **(iv)** $y^2\,dx + (x^2 - xy + y^2)dy = 0$

Q. 8 solve the differential equation $(x^2 + y^2)dy = xy\,dx$. **If y (1) = 1 and y(x_0) = e, then find the value of x_0.**

Q. 9 solve : $x\frac{dy}{dx} - y = x\sqrt{(x^2 + y^2)}$, $x > 0$

Q. 10 Solve: (i) $1 + y + x^2y)\,dx + (x + x^3)dy = 0$.

(ii)$(y + 3x^2)\,\frac{dx}{dy} = x$.

(iii) $x\,dy - (y + 2x^2)dx = 0$ **(iv)** $\frac{dy}{dx} + y\,tan\,x = 2x + x^2\,tan\,x, 0 < x < \frac{\pi}{2}$.

(v)$x\,log\,x\,\frac{dy}{dx} + y = \frac{2}{x}\,log\,x, x > 1$. **(vi)** $(1 + x - xy^2)dy = (y^3 - y)dx$

Q. 11 find the particular solution of the differential equation

$(y - sin\,x)\,dx + (tan\,x)\,dy = 0$ **satisfying the condition that y = 0 when x = 0.**

Q 12 find the particular solution of
$cos^3\,y\,dx + x\,cos\,y\,dy = sin\,y\,dy$. $when\,x = 1$ $then\,y = 0$.

KEEPING IN TOUCH

1. Is it possible to have the product of two matrices to be the null matrix while neither of them is the null matrix? If it is so, give an example.
2. If A(x1,y1), B(x2,y2) and C(x3,y3) are the vertices of an equilateral triangle with each side equal to 'a' units, prove that, $\begin{vmatrix} x1 & x2 & x3 \\ y1 & y2 & y3 \\ z1 & z2 & z3 \end{vmatrix} = \sqrt{3}\ a^2$.
3. Show that a matrix which is both symmetric and skew symmetric is a zero matrix.
4. If A + B + C = 0, then prove that $\begin{vmatrix} 1 & \cos C & \cos B \\ \cos C & 1 & \cos A \\ \cos B & \cos A & 1 \end{vmatrix} = 0$.
5. Prove tha: $\begin{vmatrix} -bc & b^2 + bc & c^2 + bc \\ a^2 + ac & -ac & c^2 + ac \\ a^2 + ab & b^2 + ab & -ab \end{vmatrix} = (ab + bc + ca)^3$.
6. Prove that: $(A^{-1})' = (A')^{-1}$. where A is an invertible matrix.
7. Two factories decided to award their emoloyees for three values(a) adaptable to new techniques(b) careful and alert in difficult situations and (c) keeping calm in tense situations, at the rate of x Rs, y Rs and z Rs per person respectively. The first factory decided to honour respectively 2, 4 and 3 employees with a total prize money of Rs 29, 000. The second factory decided to honour 5, 2 and 3 employees with the prizr money of Rs 30, 500. If three prizes per person together cost Rs 9, 500 using matrices find the award for each value.

ANSWERS

Q1. (i) 2, 1 (ii) 1, 2

Q2(i) $2xy\ y' = y^2 - x^2$ (ii) $4(2\,x\,y' - 3\,y)(y')^2 = 27x$

(iii)$(x^2 - 1)(y')^2 - 2x\,y\,y' + y^2 - 1 = 0.$

(iv) $(x^2 - y^2 - a^2)\,y' - 2x\,y = 0$

(v) $(x + y)^2((y')^2 + 1) = (x + y\,y')^2$ (vi) $\frac{dr}{dt} = -K.$

Q3. (i) $y = \sin 2x - 2\sin x - x + \log|\sec x + \tan x| + C$

(ii) $y = A\,e^{x - x^2}.$

(iii) $e^x + e^{-y} + \frac{x^3}{3} + C = 0$

(iv) $\log|\operatorname{cosec} y - \cot y| = -2\sin x + C$

Q 5. $(1 - a^2)y\,(1 + ax) = (1 + a^2)x\,(1 - ay)$

Q 6 (i) $x + y = a\tan\left(\frac{y+C}{a}\right)$

(ii) $\log|\operatorname{cosec}(x + y + \frac{\pi}{4}) - \cot(x + y + \frac{\pi}{4})| = \sqrt{2}\,x + c$

Q7. (i) $2\sqrt{\frac{x}{y}} + \log|y| = A$ (ii) $\frac{-y}{y+x} = \log|x| + C$

(iii) $\log\frac{y}{x} = Ax$ (iv) $\tan^{-1}\left(\frac{x}{y}\right) + \log|y| = C$

Q8 $x_0^2 = 3\,e^2$

Q9. $y + \sqrt{(x^2 + y^2)} = A\,x\,e^x.$

Q10. (i) $y\,x = -\tan^{-1}x + c$

(ii) $y = 3\,x^2 + C\,x$

(iii)) $y = 2\,x^2 + C\,x$

(iv) $y = x^2 + C\cos x\,.$

(v)X y log x = - 2 (1 + log x) + Cx

(vi) $x\,y = \frac{1}{2}\log\left|\frac{y-1}{y+1}\right| + C$

Q11. $y\sin x = \frac{1}{2}\sin^2 x$

Q12. $x = \tan y - 1 + 2\,e^{-\tan y}.$